

TIME TABLE

TIME	Monday	Tuesday	Wednesday	Thursday	Friday
	May 3	May 4	May 5	May 6	May 7
09.00 - 09.45	Registration	Breda	van den Berg	Mireles James	Mireles James
09.45 - 10.30	Introduction - Breda	Breda	van den Berg	Mireles James	Mireles James
11.00 - 11.45	Breden	Breden	Mireles James	van den Berg	Breden
11.45 - 12.30	Breden	Breden	Mireles James	van den Berg	Breden
14.00 - 14.45	Breda	van den Berg	Lessard	Hénot	
14.45 - 15.30	Breda	van den Berg	Lessard	Hénot	
16.00 - 16.45	Hénot	Lessard	Hénot	Lessard	
16.45 - 17.30	Hénot	Lessard	Hénot	Lessard	
	welcome aperitif		social dinner		

ADMISSION AND ACCOMMODATION

The course is offered in a hybrid format, allowing participants the flexibility to attend either in person or remotely via the Microsoft Teams platform. Admission to on-site attendance is granted on a first-come, first-served basis to comply with the capacity of the lecture room.

Registration fees:

- **Early Bird On-Site Participation: € 650.00 + VAT***

Deadline: March 3, 2027

- **Late On-Site Participation: € 800.00 + VAT***

Deadline: April 21, 2027

- **Live Streaming Online Participation: € 250.00 + VAT***

Deadline: April 21, 2027

On-site participation includes a complimentary bag, five fixed menu buffet lunches, hot beverages, downloadable lecture notes.

Online participation includes downloadable lecture notes.

Application forms should be submitted online through the website: <http://www.cism.it>. A confirmation message will be sent to participants whose applications are accepted.

Upon request, and subject to availability, a limited number of on-site participants can be accommodated at the CISM Guest House for € 35 per person per night. To request accommodation, please contact: foresteria@cism.it

* where applicable; bank charges are not included - Italian VAT is 22%.

CANCELLATION POLICY

Applicants may cancel their registration and receive a full refund by notifying the CISM Secretariat in writing (via email) no later than:

- March 3, 2027, for early bird on-site participation;
- April 3, 2027, for late on-site participation;
- April 21, 2027, for online participation.

No refunds after the deadlines. Cancellation requests received before these deadlines and incorrect payments will be subject to a € 50.00 handling fee.

CISM GRANTS

A limited number of participants from universities and research centers who do not receive support from their institutions can request a waiver of the registration fee and/or free lodging.

Requests must be submitted via the CISM website by **March 3, 2027**. Applicants should select the relevant option in the online application form and provide a curriculum vitae and a letter of recommendation from their department head or supervisor confirming that their institution is unable to provide financial support. Preference will be given to applicants from countries that sponsor CISM.

For further information please contact:

CISM (Seat of the course)
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e-mail: info@cism.it | www.cism.it



COMPUTER-ASSISTED PROOFS IN DELAYED AND INFINITE-DIMENSIONAL DYNAMICAL SYSTEMS

CISM Advanced School
coordinated by

Dimitri Breda
CDLab - University of Udine, Italy

Jean-Philippe Lessard
McGill University, Montreal, Canada

Udine May 3 - 7 2027

COMPUTER-ASSISTED PROOFS IN DELAYED AND INFINITE-DIMENSIONAL DYNAMICAL SYSTEMS

Computer-assisted proofs (CAPs), also known as validated or rigorous numerics, have become a powerful methodology for turning numerical evidence into mathematical theorems. Recent advances in combining tools from nonlinear analysis, spectral methods, interval arithmetic, and high-performance computing now make it possible to rigorously prove existence, uniqueness and qualitative properties of solutions in ordinary, delay and partial differential equations, including equilibria, periodic orbits, invariant manifolds, connecting orbits and chaotic dynamics.

This CISM advanced course will provide a coherent introduction to modern CAP techniques for delayed and infinite-dimensional dynamical systems. It will guide participants from the basic principles of a posteriori validation to state-of-the-art methods for

delay differential equations and evolution PDEs. A central theme will be the rigorous validation paradigm: compute a high-quality numerical approximation, embed it in a suitable Banach-space formulation, and prove, by explicit estimates and Newton-Kantorovich or contraction arguments, that a true solution exists nearby.

General outline

The course will be organized in six complementary lecture series combining theory, proof strategies and computational demonstrations. Topics will include: foundations of validated nonlinear analysis; semigroup and functional-analytic approaches to delay equations; Fourier and Chebyshev techniques and coefficient-space methods; rigorous eigenvalue and stability computations; validated computation of invariant manifolds

and connecting orbits; and rigorous time-stepping for PDE-grounded infinite-dimensional dynamics. Particular attention will be paid to closing estimates: choosing effective norms, controlling finite-dimensional truncations and infinite-dimensional tails, bounding nonlinear terms, and managing rounding and discretization errors.

The course will also introduce modern software workflows for rigorous computation, including interval arithmetic tools and Julia-based implementations such as `RadiiPolynomial.jl`. Laboratory activities and demonstrations will show how theoretical ingredients - radii polynomials, validated Newton steps, operator bounds and tail estimates - translate into implementable routines. Concrete applications will include, e.g., celestial mechanics (3- and N-body problems), fluid dynamics

(Navier-Stokes), geodesic flows on manifolds and chaotic dynamics in general. Participants will be asked to bring their own laptop.

Target audience

The course is mainly addressed to graduate students, postdoctoral researchers and early-career scientists in Mathematics, Engineering, Physics and related fields concerned with dynamical systems, computational methods, delay equations and PDE models. It may also interest researchers working in applications such as biology, population dynamics and economics. Starting from core principles, the lectures will be accessible to participants with some background in analysis, dynamical systems and numerical methods, while also providing a roadmap to current research frontiers and open problems.

INVITED LECTURERS

Jan Bouwe van den Berg - VU University Amsterdam, The Netherlands

5 lectures plus discussion on:

periodic solutions in differential equations, Newton-Kantorovich in infinite dimensions, Banach algebras of sequences with exponential decay, analyticity, Fourier series and connection to Taylor/Chebyshev series, finite-dimensional projection and numerics.

Dimitri Breda - CDLab, University of Udine, Italy

Introduction plus 4 lectures on:

well-posedness, representation of delay equations and infinite-dimensional dynamical systems; semigroup theory, spectral analysis; equilibria, periodic orbits, chaos; bifurcation analysis; pseudospectral methods, examples from engineering, population dynamics.

Maxime Breden - École Polytechnique, Paris-Saclay, France

5 lectures plus discussion on:

computer-assisted proofs in finite dimensions, interval arithmetic, Newton-Kantorovich theorem, periodic orbits and chaos in discrete dynamical systems, rigorous enclosure of eigenvalues, extensions to PDEs: approximation theory and compactness estimates.

Olivier Hénot - National Taiwan University, Taiwan

5 lectures plus discussion on:

introduction to Julia programming, the libraries `IntervalArithmetic.jl` (for interval arithmetic), and `RadiiPolynomial.jl` (handles Fourier, Taylor and Chebyshev bases, convolution, differential, and integral operators), examples, connecting orbits and chaos.

Jean-Philippe Lessard - McGill University, Montreal, Canada

5 lectures plus discussion on:

initial value problems in delay equations, explicitly and implicitly defined step map, discretization via Chebyshev series, discrete smoothing, infinite-dimensional compact discrete dynamical systems, dynamics, boundary value problems, connecting orbits.

Jason D. Mireles James - Florida Atlantic University, USA

5 lectures plus discussion on:

parameterization method for un/stable manifolds of fixed points and periodic orbits of maps and ODEs, stability of equilibria in DDEs, Morse indices, parameterization method for unstable manifolds in DDEs, graph transform approach for stable manifolds in DDEs.

PRELIMINARY SUGGESTED READINGS

J.B. van den Berg and J.-P. Lessard. Rigorous numerics in dynamics, *Notices of the AMS*, 62(9): 1057–1061, 2015.

M. Breden. Computer-assisted proofs for differential equations and dynamical systems, Institut Polytechnique de Paris, 2025.

D. Breda. Pseudospectral methods for the stability analysis of delay equations. Part I: The infinitesimal generator approach,

in D. Breda (Ed.), *Controlling Delayed Dynamics: Advances in Theory, Methods and Applications*, CISM Lecture Notes (pp. 65–94). Wien-New York: Springer, 2023.

J. Gómez-Serrano. Computer-assisted proofs in PDE: a survey. *SeMA J.* 76(3): 459–484, 2019.

O. Hénot. `RadiiPolynomial.jl`, <https://github.com/OlivierHnt/RadiiPolynomial.jl>, 2021.

J.-P. Lessard. Continuation of solutions and studying delay differential equations via rigorous numerics, in *Rigorous numerics in dynamics*, 81–122, *Proc. Sympos. Appl. Math.*, 74, Amer. Math. Soc., Providence, RI, 2018.

J.-P. Lessard and J.D. Mireles James. A rigorous implicit C^1 Chebyshev integrator for delay equations. *J. Dyn. Differ. Equ.*, 33(4): 1959–1988, 2021.

M.T. Nakao, M. Plum and Y. Watanabe. Numerical verification methods and computer-assisted proofs for partial differential equations, *Springer Ser. Comput. Math.*, 53, Springer, Singapore, 2019.

W. Tucker. Validated numerics. A short introduction to rigorous computations. Princeton University Press, Princeton, NJ, 2011.

LECTURES

All lectures will be given in English. Lecture notes can be downloaded from the CISM web site. Instructions will be sent to accepted participants.