

## DERIVED CATEGORIES OF FANO VARIETIES, HYPERKÄHLER VARIETIES AND THEIR PERIOD MAPS

The aim of this school is to introduce young researchers and PhD students to the theory of K3 surfaces, their higher-dimensional analogues, hyperkähler manifolds and the related theory of derived categories of Fano varieties. These manifolds are interesting from several points of view: dynamical (some have interesting automorphism groups), arithmetical, and geometric. It is also one of those rare cases where the Torelli theorem allows for a powerful link between the geometry of these manifolds and lattice theory. Our aim is more to provide, for specific families of hyperkähler manifolds (which are projective deformations of punctual Hilbert schemes of K3 surfaces), a panorama of results about projective embeddings, automorphisms, moduli spaces, period maps and domains, rather than a complete reference guide. Moreover, the derived category counterpart of the above theory will also be presented.

For further information about the school please refer to <http://derived.dmif.uniud.it/>

### LECTURERS

#### OLIVIERRE DEBARRE

Université Paris Diderot (Paris 7), France  
5 lectures on:  
Fano varieties, Hyperkähler varieties, and their period maps

#### ALEXANDER KUZNETSOV

Algebra Section, Steklov Mathematical Institute - Russian Academy of Sciences, Moscow, Russia  
5 lectures on:  
Derived categories of Fano varieties

#### GIOVANNI MONGARDI

Alma Mater Studiorum - University of Bologna, Italy  
Assistant to the course:  
Fano varieties, Hyperkähler varieties, and their period maps

#### ALEXANDER PERRY

School of Mathematics Institute for Advanced Study  
Columbia University, NY, USA  
Assistant to the course:  
Derived categories of Fano varieties

## ADMISSION AND ACCOMMODATION

The registration fee is 50,00 Euro + VAT taxes\*, where applicable (bank charges are not included).

The registration fee includes five fixed menu buffet lunches, coffee breaks, downloadable lecture notes, wi-fi internet access and a complimentary bag.

A limited number of participants can be accommodated at CISM Guest House at the price of 30 Euro per person/night.

Seven single and eight double rooms (to be shared with a colleague) are available, and they will be assigned following the order of reservation requests.

At the following link you can find a list of hotels with special prices for CISM participants: [www.cism.it/en/contact-us/hotels/](http://www.cism.it/en/contact-us/hotels/)  
Booking should be made directly with the hotels.

Applications should be made on-line through our web site: [www.cism.it/en/activities/courses/E2102](http://www.cism.it/en/activities/courses/E2102)

A message of confirmation will be sent to accepted participants. Information about travel and accommodation is available on our web site, or can be mailed upon request.

Applicants may cancel their course registration and receive a full refund by notifying CISM Secretariat in writing (by email) no later than two weeks prior to the start of the course.

If cancellation occurs less than two weeks prior to the start of the course, the registration fee will be retained.

\* Italian VAT is 22%

For further information please contact:

#### CISM

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ACADEMIC YEAR 2021  
University of Udine  
International Centre for Mechanical Sciences



UNIVERSITÀ  
DEGLI STUDI  
DI UDINE  
hic sunt futura



Centro Internazionale  
di Scienze Meccaniche  
International Centre  
for Mechanical Sciences

## DERIVED CATEGORIES OF FANO VARIETIES, HYPERKÄHLER VARIETIES AND THEIR PERIOD MAPS

CISM-UniUD Joint  
course  
coordinated by

Pietro Corvaja, Pietro De Poi, Francesco Zucconi  
University of Udine, Italy

Ugo Bruzzo  
SISSA, Trieste, Italy



Advances in Moduli Theory and  
Birational Classification  
A PRIN2017 network

Udine November 8 - 12 2021

## PROGRAMME AND LECTURERS

**Olivier Debarre**

assistant: **Giovanni Mongardi**

**Fano varieties, Hyperkähler varieties, and their period maps**

The period point of a smooth projective complex variety is the Hodge structure on one of its cohomology groups, considered as a point in some flag variety (called the period domain). The period map associates with the variety (seen as a point in some moduli space) its period point (seen as a point in the period domain). The period map is holomorphic but can be very complicated if the Hodge structure has many pieces, so we will stick to the simplest cases, where the period domain is algebraic and has a relatively simple structure:

Hodge structures of weight one: this is the case when the Hodge decomposition has only two pieces and defines an abelian variety. This happens for curves, for Fano threefolds, and for cubic hypersurfaces and Gushel-Mukai varieties of dimension 5. The period domain is the Siegel upper half space, a bounded symmetric domain of type III.

Hodge structures of K3 type: this is the case when the Hodge decomposition has only three pieces, two of them having dimension 1. This happens for K3 surfaces (hence the name), Hyperkähler varieties, cubic hypersurfaces and Gushel-Mukai varieties of dimension 4. The period domain is an open subset of a quadratic hypersurface, a bounded symmetric domain of type IV.

The Torelli theorem asserts that a smooth projective curve is determined by its Jacobian. This is equivalent to saying that (some form of) their period map is injective. We will study the geometrical properties of the period maps in some of the cases mentioned above: cubic hypersurfaces, Gushel-Mukai varieties, and hyperkähler varieties. In several cases, one can associate with a Fano variety with a Hodge structure of K3 type a hyperkähler variety with isomorphic Hodge structure (same period point). We will explore some of these geometrical constructions.

**Alexander Kuznetsov**

assistant: **Alex Perry**

**Derived categories of Fano varieties**

The derived category of coherent sheaves is the main homological invariant of an algebraic variety. Semiorthogonal decompositions of derived categories provide in many cases (mostly for Fano varieties) unexpected relations between their geometry and that

of other varieties. We will discuss some of these relations, mostly in dimensions 3 and 4. First, we will discuss Fano threefolds whose derived categories contain derived categories of curves as components. We will see the relevance of these curves for the birational geometry of these varieties (including the case of non-closed fields) and how these curves appear in the study of moduli spaces of sheaves. Second, we will discuss examples of different Fano threefolds sharing non-trivial components in their derived categories. Again, we will see how this affects the geometry of the varieties involved. Finally, we will discuss examples of varieties of dimension 4 and higher (cubic fourfolds, Gushel-Mukai fourfolds and sixfolds, Debarre-Voisin 20-folds) whose derived categories contain a non-commutative K3 surface as a component. In the case of fourfolds, the properties of these components are expected to be directly related to the birational geometry of these varieties and the existence of hyperkähler moduli spaces of sheaves on them

## SCHEDULE

### *Monday, November 8*

**9:00 - 10:45 Registration**

**11:00 - 12:30 Olivier Debarre**

Fano varieties, Hyperkähler varieties, and their period maps

**12:30 - 14:30 Lunch**

**14:30 - 16:30 Alexander Kuznetsov**

Derived categories of Fano varieties

### *Tuesday, November 9*

**9:00 - 10:30 Olivier Debarre**

Fano varieties, Hyperkähler varieties, and their period maps

**10:30 - 11:00 Coffee break**

**11:00 - 12:30 Alexander Kuznetsov**

Derived categories of Fano varieties

**12:30 - 15:00 Lunch**

**15:00 - 16:30 Exercise class by Giovanni Mongardi**

**16:30 - 17:00 Coffee break**

**17:00 - 18:30 Exercise class by Alex Perry**

### *Wednesday, November 10*

**09:00 - 10:30 Olivier Debarre**

Fano varieties, Hyperkähler varieties, and their period maps

**10:30 - 11:00 Coffee break**

**11:00 - 12:30 Alexander Kuznetsov**

Derived categories of Fano varieties

**12:30 - 15:00 Lunch**

**15:00 - 16:30 Exercise class by Giovanni Mongardi**

**16:30 - 17:00 Coffee break**

**17:00 - 18:30 Exercise class by Alex Perry**

**20:00 - 21:30 Social dinner**

### *Thursday, November 11*

**11:00 - 12:30 Olivier Debarre**

Fano varieties, Hyperkähler varieties, and their period maps

**12:30 - 13:30 Lunch**

**13:30 - 15:00 Alexander Kuznetsov**

Derived categories of Fano varieties

**15:00 - 20:00 Free afternoon**

### *Friday, November 12*

**9:00 - 10:00 Olivier Debarre**

Fano varieties, Hyperkähler varieties, and their period maps

**10:00 - 10:30 Coffee break**

**11:00 - 12:30 Alexander Kuznetsov**

Derived categories of Fano varieties

**12:30 - 15:00 Lunch**

**15:00 - 16:30 Exercise class by Giovanni Mongardi**

**16:30 - 17:00 Coffee break**

**17:00 - 18:30 Exercise class by Alex Perry**