

## TIME TABLE

TIME	Monday October 21	Tuesday October 22	Wednesday October 23	Thursday October 24	Friday October 25
09.00 - 09.45	Registration	Spillane	Rheinbach	Gosselet	Gosselet
09.45 - 10.30	Klawonn	Rheinbach	Klawonn	Klawonn	Spillane
11.00 - 11.45	Gosselet	Klawonn	Spillane	Rheinbach	Rheinbach
11.45 - 12.30	Gosselet	Knepley	Kučera	Knepley	Knepley
14.00 - 14.45	Spillane	Gosselet	Kučera	Kučera	
14.45 - 15.30	Kučera	Kučera	Gosselet	Spillane	
16.00 - 16.45	Knepley	Poster Session	Knepley	Klawonn	
16.45 - 17.30	Rheinbach	Poster Session	Knepley	Round table	
18.00	Welcome aperitif				

### ADMISSION AND ACCOMMODATION

The course is offered in a hybrid format giving the possibility to attend the course also by remote (on Microsoft Teams platform). On-site places are limited and assigned on first come first served basis. The registration fees are:

#### - On-site participation, 600.00 Euro + VAT\*

This fee includes a complimentary bag, five fixed menu buffet lunches, hot beverages, downloadable lecture notes. Deadline for on-site application is September 21, 2024.

#### - Live Streaming Online participation, 250.00 Euro + VAT\*

This fee includes downloadable lecture notes. Deadline for online application is October 9, 2024. Application forms should be sent on-line through the following web site: <http://www.cism.it>

A message of confirmation will be sent to accepted participants.

Upon request a limited number of on-site participants can be accommodated at CISM Guest House at the price of 35 Euro per person/night (mail to: [foresteria@cism.it](mailto:foresteria@cism.it)).

*\* where applicable (bank charges are not included)  
Italian VAT is 22%.*

### CANCELLATION POLICY

Applicants may cancel their registration and receive a full refund by notifying CISM Secretariat in writing (by email) no later than:

- September 21, 2024 for on-site participants (no refund after the deadline);

- October 9, 2024 for online participants (no refund after the deadline).

Cancellation requests received before these deadlines will be charged a 50.00 Euro handling fee. Incorrect payments are subject to Euro 50,00 handling fee.

### GRANTS

A limited number of participants from universities and research centres who are not supported by their own institutions can request the waiver of the registration fee and/or free lodging.

Requests should be sent to CISM Secretariat by **August 21, 2024** along with the applicant's curriculum and a letter of recommendation by the head of the department or a supervisor confirming that the institute cannot provide funding. Preference will be given to applicants from countries that sponsor CISM.

*For further information please contact:*

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# DOMAIN DECOMPOSITION AND MULTIGRID METHODS FOR COMPUTATIONAL STRUCTURE MECHANICS

Advanced School  
coordinated by

**Pierre Gosselet**  
LaMCube, Université Lille, CNRS  
Centrale Lille, France

**Udine October 21 - 25 2024**

# DOMAIN DECOMPOSITION AND MULTIGRID METHODS FOR COMPUTATIONAL STRUCTURE MECHANICS

The increased sophistication of materials (e.g., composites, 3D printed microstructures) and the extension of their range of use (e.g., complex triaxial state of stress induced by complex geometry) threaten to render obsolete the traditional “rules of thumb” used by designers and issued from decades of experience. Simple engineering optimization approaches are no longer applicable. Moreover, the complexity of failure scenarios makes it difficult to predict accurately the resilience of structures which is crucial when aiming at virtual testing and certification.

The solution to these difficulties lies in the ability to conduct reliable simulations at the scale of the whole structure. In this context, reliability implies to be able to use fine scale models that accurately reproduce nonlinear critical phenomena like plasticity,

cracking, delamination. This in turn implies to use fine adapted meshes potentially containing billions of degrees of freedom.

Modern cluster hardware made out of thousands of multicore processors connected by a fast network makes it in theory possible to bear such a computational load. But only few solvers are capable of correctly exploiting these resources. Indeed, problems from structure mechanics possess many unfavorable features associated with the use of unstructured finite element discretization, the presence of strong heterogeneity, the history-dependent nonlinearity (elastoviscoplasticity, damage, cracking). The mechanical engineering community is deeply influenced by legacy commercial software which relies on direct solvers which, in spite of their impressive recent progress in terms of parallelism, are not

able to scale up to the desired complexity.

However, domain decomposition and multigrid methods have been the object of important studies and developments in the last 30 years, and thanks to recent improvement regarding their robustness, they have reached a remarkable level of maturity. In addition to their strong mathematical foundations, these methods possess essential mechanical interpretations and offer new modeling opportunities (e.g., non-overlapping domain decomposition for the contact between subdomains, multigrid methods to handle nested models).

This course aims at presenting state-of-the-art domain decomposition and multigrid solvers for solid mechanics. The course will balance theoretical and practical aspects, as well as mathematical and mechanical points of view.

The course will also discuss the open source software libraries which propose implementations of the methods.

The course is intended for graduate students working in computational structure mechanics, numerical analysis of partial differential equations, and high-performance computing. The content will also be very appealing for mechanical or civil engineers from industry seeking high performance, where the characteristic size of critical phenomena is very small compared to the structure. This framework encompasses composite structures (aerospace industry), microstructure-driven metallic parts (rolling elements in trains and cars), reinforced concrete buildings, biomedical applications...

## PRELIMINARY SUGGESTED READINGS

A. Toselli and O. Widlund. 2005. Domain decomposition methods – algorithms and theory. Springer series in computational mathematics.

P. E. Farrell, M. G. Knepley, L. Mitchell, F. Wechsung. 2021. PCPATCH: Software for the Topological Construction of Multigrid Relaxation Methods. ACM Trans. Math. Softw. 47, 3, Article 25.

D. A. May, P. Sanan, K. Rupp, M. G. Knepley, B. F. Smith, 2016. Extreme-Scale Multigrid Components within PETSc.

Z. Dostál, D. Horák, R. Kučera. 2006. Total FETI – an easier implementable variant of the FETI method for numerical solution of elliptic PDE. Comm. Numer. Meth. Engng. 22(12).

R. Kučera, J. Machalová, H. Netuka, P. Ženčák. 2013. An interior-point algorithm for the minimization arising from 3D contact problems with friction. Optimization Methods and Software, 28:6.

N. Spillane, V. Dolean, P. Hauret, F. Nataf, C. Pechstein, R. Scheichl. 2014. Abstract robust coarse spaces for systems of PDEs via

generalized eigenproblems in the overlaps. Numerische Mathematik, 126(4).

C. Bovet, A. Parret-Fréaud, N. Spillane, P. Gosselet. Adaptive multipreconditioned FETI: scalability results and robustness assessment. Computers & Structures, 2017, 193, pp.1-20.

A. Heinlein, O. Rheinbach, and F. Röver. 2023. Parallel Scalability of Three-Level FROSch Preconditioners to 220 000 Cores using the Theta Supercomputer. SIAM J. Sci. Comput., 45(3):S173-S198.

A. Klawonn, M. Kuhn, O. Rheinbach. 2016. Adaptive coarse spaces for FETI-DP in three dimensions. SIAM Journal on Scientific Computing 38 (5), A2880-A2911

V. Rey, C. Rey, P. Gosselet. 2014. A strict error bound with separated contributions of the discretization and of the iterative solver in non-overlapping domain decomposition methods. 2014. Computer Meth, in App, Mech, and Engng. 270, 293-303.

## INVITED LECTURERS

**Pierre Gosselet** - LaMCube, Université Lille, CNRS, Centrale Lille, France

*6 lectures on:* Mechanical introduction to non-overlapping domain decomposition methods, discussion about classical preconditioning and coarse space, extension to nonlinear problems (in particular elastoviscoplasticity), link with verification and error estimation, global/local coupling and asynchronous computing, illustration on simplified examples from aeronautic applications.

**Axel Klawonn** - University of Cologne, Köln, Germany  
*5 lectures on:* Introduction to nonoverlapping DDM using Lagrange multipliers; additional constraints for parallel scalability; theoretical considerations and condition number estimates; heterogeneous problems and adaptive coarse spaces; nonlinear domain decomposition methods; nonlinear elimination; domain decomposition and machine learning; application to problems in solid mechanics.

**Matthew Knepley** - University at Buffalo, NY, USA  
*6 lectures on:* Multigrid methods, including robust smoothers for operators with large kernels, parallel considerations, and monolithic multigrid for systems of equations. Hands-on introduction to the PETSc library focusing on scalable multigrid solvers and parallel implementations. Applications to geodynamics.

**Radek Kučera** - Technical University of Ostrava, Ostrava-Poruba, Czech Republic  
*5 lectures on:* Total FETI for linear elasticity, computation of generalized inverses, algorithms for contact problems based on minimization with constraints (simple bounds, quadratic constraints, successive iterations for Coulomb friction). Contact problems with TFETI, other algorithms (interior point method and semi-smooth Newton method), extension to contact conditions in CFD.

**Oliver Rheinbach** - TU Bergakademie Freiberg, Germany  
*5 lectures on:* Introduction to overlapping domain decomposition methods (DDM) with exotic coarse spaces; theory of overlapping DDM with GDSW and related coarse spaces; improving the range of scalability by additional levels; nonlinear overlapping DDM; the open source FROSch framework as part of the Trilinos software library and application of FROSch to problems in chemomechanics.

**Nicole Spillane** - CNRS, CMAP, Ecole Polytechnique, Palaiseau, France  
*5 lectures on:* Krylov subspace methods for solving linear systems (conjugate gradient, GMRES...), acceleration by preconditioning and deflation, convergence theory, abstract presentation of domain decomposition methods (emphasis on Additive Schwarz, BDD and FETI), GenEO coarse spaces for robustness and scalability, multipreconditioning, non-symmetric problems.

## LECTURES

All lectures will be given in English. Lecture notes can be downloaded from the CISM web site. Instructions will be sent to accepted participants.